Calculus and Vectors, Grade 12, University Preparation

Course Title: Calculus and Vectors
Course Code: MCV4U
Grade: 12
Course Type: University Preparation
Credit Value: 1.0
Prerequisites: Advanced Functions (MHF4U), but it may be taken concurrently.

Curriculum Document: Mathematics, The Ontario Curriculum, Grades 11 and 12, 2007 (Revised)
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Teacher(s):

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Course Description:

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

Unit Title and Description	Time Allocated
Vectors There are four main topics pursued in this initial unit of the course. These topics are: an introduction to vectors and scalars, vector properties, vector operations, and plane figure properties. Students will tell the difference between a scalar and vector quantity, they will represent vectors as directed line segments and perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. Students will conclude the first half of the unit by proving some properties of plane figures, using vector methods and by modeling and solving problems involving force and velocity. Next, students learn to represent vectors as directed line segments and to perform the operations of addition, subtraction, and scalar multiplication on geometric vectors with and without dynamic geometry software. The final topic involves students in proving some properties of plane figures using vector methods.	12 Hours

Linear Dependence and Coplanarity Cartesian vectors are represented in two-space and three-space as ordered pairs and triples, respectively. The addition, subtraction, and scalar multiplication of Cartesian vectors are all investigated in this unit. Students investigate the concepts of linear dependence and independence, and collinearity and coplanarity of vectors.	10 Hours
Vector Applications Applications involving work and torque are used to introduce and lend context to the dot and cross products of Cartesian vectors. The vector and scalar projections of Cartesian vectors are written in terms of the dot product. The properties of vector products are investigated and proven. These vector products will be revisited to predict characteristics of the solutions of systems of lines and planes in the intersections of lines and planes.	10 Hours
Intersection of Lines and Plans This unit begins with students determining the vector, parametric and symmetric equations of lines in R2 and R3. Students will go on to determine the vector, parametric, symmetric and scalar equations of planes in 3-space. The intersections of lines in 3-space and the intersections of a line and a plane in 3-space are then taught. Students will learn to determine the intersections of two or three planes by setting up and solving a system of linear equations in three unknowns. Students will interpret a system of two linear equations in two unknowns geometrically, and relate the geometrical properties to the type of solution set the system of equations possesses. Solving problems involving the intersections of lines and planes, and presenting the solutions with clarity and justification forms the next challenge.	12 Hours
Concepts of Calculus A variety of mathematical operations with functions are needed in order to do the calculus of this course. This unit begins with students developing a better understanding of these essential concepts. Students will then deal with rates of change problems and the limit concept. While the concept of a limit involves getting close to a value but never getting to the value, often the limit of a function can be determined by substituting the value of interest for the variable in the function. Students will work with several examples of this concept. The indeterminate form of a limit involving factoring, rationalization, change of variables, and one-sided limits are all included in the exercises undertaken next in this unit. To further investigate the concept of a limit, the unit briefly looks at the relationship between a secant line and a tangent line to a curve. To this point in the course, students have been given a fixed point and have been asked to find the tangent slope at that value. In this section of the unit, students will determine a tangent slope function similar to what they had done with a secant slope function. Sketching the graph of a derivative function is the final skill and topic.	12 Hours
Derivatives	12 Hours

The concept of a derivative is, in essence, a way of creating a short cut to determine the tangent line slope function that would normally require the concept of a limit. Once patterns are seen from the evaluation of limits, rules can be established to simplify what must be done to determine this slope function. This unit begins by examining those rules including: the power rule, the product rule, the quotient rule, and the chain rule followed by a study of the derivatives of composite functions. The next section is dedicated to finding the derivative of relations that cannot be written explicitly in terms of one variable. Next, students will simply apply the rules they have already developed to find higher order derivatives. As students saw earlier, if given a position function, they can find the associated velocity function by determining the derivative of the position function. They can also take the second derivative of the position function and create a rate of change of velocity function that is more commonly referred to as the acceleration function which is where this unit ends.	
Curve Sketching In previous math courses, functions were graphed by developing a table of values and smooth sketching between the values generated. This technique often hides key detail of the graph and produces a dramatically incorrect picture of the function. These missing pieces of the puzzle can be found by the techniques of calculus learned thus far in this course. The key features of a properly sketched curve are all reviewed separately before putting them all together into a full sketch of a curve.	12 Hours
Derivative Applications and Related Rates A variety of types of problems exist in this unit and are generally grouped into the following categories: Pythagorean Theorem Problems (these include ladder and intersection problems), Volume Problems (these usually involve a 3-D shape being filled or emptied), Trough Problems, Shadow problems, and General Rate Problems. During this unit, students will look at each of these types of problems individually.	9 Hours
Derivatives of Exponents and Logarithmic Functions This unit begins with examples and exercises involving exponential and logarithmic functions using Euler's number (e). But as students have already seen, many other bases exist for exponential and logarithmic functions. Students will now look at how they can use their established rules to find the derivatives of such functions. The next topic should be familiar as the steps involved in sketching a curve that contains an exponential or logarithmic function are identical to those taken in the curve sketching unit studied earlier in the course. Because the derivatives of some functions cannot be determined using the rules established so far in the course, students will need to use a technique called logarithmic differentiation which is introduced next.	9 Hours
Trigonometric Differentiation and Applications A brief trigonometry review kicks off this unit. Then, students turn their attention to special angles and the CAST rule which has been developed to identify which of the basic trigonometric ratios is positive and negative in the four quadrants. Students will then solve trigonometry equations using the CAST rule to locate other solutions. Two fundamental trigonometric limits are investigated for the concepts of trigonometric calculus to be fully understood. The unit ends, as in all other units in the course, with an assignment and a unit quiz.	9 Hours
Final Assessment	3 Hours

The final assessment for this course consists of a final exam that is worth 30% of a student's overall grade.	

Overall Curriculum Expectations
Rate of Change 1. demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit; 2. graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative; 3. verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.
Derivatives and their Applications 1. make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching; 2. solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.
Geometry and Algebra of Vectors 1. demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications; 2. perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications; 3. distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space; 4. represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.

Resources Required:

This course is entirely online and does not require nor rely on any textbook. The materials required for the course are:

- A scanner, smart phone camera, or similar device to digitize handwritten or hand-drawn work,
- A non-programmable, non-graphing, scientific calculator.
- Spreadsheet software
- Word processing software

- Graphing software (available online through Desmos)

Teaching and Learning Strategies:

The overriding aim of this course is to help students use the language of mathematics skillfully, confidently, and flexibly. A wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests, and ability levels. The following mathematical processes are used throughout the course as strategies for teaching and learning the concepts presented:

- *Problem solving*: This course scaffolds learning by providing students with opportunities to review and activate prior knowledge, and build off of this knowledge to acquire new skills. The course guides students toward recognizing opportunities to apply knowledge they have gained to solve real-world mathematics problems relating to careers that require mathematics.
- *Activating*: This course scaffolds learning by providing students with opportunities to review and activate prior knowledge, and build off of this knowledge to acquire new skills. The course guides students toward recognizing opportunities to apply knowledge they have gained to solve real-world mathematics problems relating to careers that require mathematics.
- *Connecting*: The course activates prior knowledge when introducing a new concept in order to make a smooth connection between previous learning and new concepts, and introducing skills in context to make connections between particular manipulations and problems that require them.
- *Representing*: Through the use of examples, practice problems, and solution videos, the course models various ways to demonstrate understanding, poses questions that require students to use different representations as they are working at each level of conceptual development - concrete, visual or symbolic, and allows individual students the time they need to solidify their understanding at each conceptual stage.
- *Selecting Tools and Computational Strategies*: This course models the use of graphing software to help solve problems and to familiarize students with technologies that can help make solving problems faster and more accurate. Students will also investigate software used in a career that they are interested in, and describe how it relates to mathematics.
- *Connecting*: This course connects the concepts taught to real-world applications (e.g. concepts taught in geometry are related to careers in manufacturing). Students will have opportunities to connect previous concepts to new concepts through posed problems, investigations, and enrichment activities.
- *Self-Assessment*: Through the use of interactive activities (e.g. multiple choice quizzes, and drag-and-drop activities) students receive instantaneous feedback and are able to self-assess their understanding of concepts.

Assessment and Evaluation Strategies

Every student attending Christian Virtual School is unique. We believe each student must have the opportunities to achieve success according to their own interests, abilities, and goals. Like the Ministry of Education, we have defined high expectations and standards for graduation, while

introducing a range of options that allow students to learn in ways that suit them best and enable them to earn their diplomas. Christian Virtual School's Assessment, Evaluation, and Reporting Policy is based on seven fundamental principles, as outlined in the Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools document.

When these seven principles are fully understood and observed by all teachers, they guide the collection of meaningful information that helps inform instructional decisions, promote student engagement, and improve student learning. At Christian Virtual School, teachers use practices and procedures that:

- are fair, transparent, and equitable for all students;
- support all students, including those with special education needs, those who are learning English, and those who are First Nation, Métis, or Inuit;
- are carefully planned to relate to the curriculum expectations and learning goals and, as much as possible, to the interests, learning styles and preferences, needs, and experiences of all students;
- are communicated clearly to students and parents or guardians at the beginning of the school year or course and at other appropriate points throughout the school year or course;
- are ongoing, varied in nature, and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning;
- provide ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement; and
- develop students' self-assessment skills to enable them to access their own learning, set specific goals, and plan next steps for their learning.

For more information on Christian Virtual School's general assessment and evaluation strategies, you can refer to our Assessment, Evaluation, and Reporting Policy.

To ensure that we are meeting the principles of *Growing Success*, we carefully plan all the assessments within our courses.

First and foremost, they are designed as opportunities for students to improve their learning. Assessment for the purpose of improving student learning is seen as both "assessment for learning" and "assessment as learning," according to *Growing Success*. As part of assessment for learning, teachers provide students with descriptive feedback and coaching for improvement. Teachers engage in assessment as learning by helping all students develop their capacity to be independent, autonomous learners who can set individual goals, monitor their own progress, determine next steps, and reflect on their thinking and learning. Examples of these types of assessments in this course include:

Assessment for Learning	Assessment as Learning
Discussion activities	Reflection activities
Practice quizzes	Self-directed exercises
Practice assignments	

Second, we focus on a balance between assessing students' acquisition of knowledge as well as

their skills of thinking, communication, and application of subject-specific material. In this course, you can expect assessment to be divided into the following balance:

Percentage	Skill
20	Knowledge and Understanding: Subject-specific content acquired and the comprehension of its meaning and significance
30	Thinking/Inquiry: The use of critical and creative thinking skills and/or processes
20	Communication: The conveying of meaning through various forms
30	Application: The use of knowledge and skills to make connections within and between various contexts

Lastly, the assessments are designed so that teachers have an opportunity to gain an understanding of a student's learning through direct observation of students, one-on-one conversations with students, and evaluating products that students submit. Examples of these methods in this course include:

Observation	Conversation	Product
Practice assignments	Interaction through emails	Unit tests
Practice quizzes	Teacher-student discussions	Problem sets
		Graphing activities

Program Planning Considerations

Each of our courses have been designed by a team of educators to create an environment infused with creativity, flexibility, choice, and variety, with the goal to help every student succeed. We also take into consideration several topics that span disciplines and ensure we incorporate these into each of our courses.

Program Planning Considerations
Students with Special Needs Christian Virtual School is committed to ensuring that all students are provided with the learning opportunities and supports they require to succeed. Our courses are made to offer flexible, personalized learning experiences. By maintaining an asynchronous model, students can move through their courses at their own pace, ensuring they are able to take the time they need to understand concepts or work with their teacher if they hit roadblocks. Christian Virtual School courses also incorporate choice, allowing students to submit work in a variety of mediums or formats to communicate their ideas. In addition to the flexibility built into the courses, Christian Virtual School will implement the accommodations that are listed in a student's Individual Education Plan (IEP) that are applicable to the online learning environment. In these cases, the learning expectations will be the same as or similar to the expectations outlined in the curriculum document but supports will be provided to help students achieve those expectations. Common accommodations in the environment are reducing the workload, simplifying tasks and materials, providing extra time for tests and exams, allowing scribing or the use of specialized equipment, and not deducting marks for spelling.

English Language Learners

Although all our courses are only offered in English at this time, Christian Virtual School welcomes students learning the English language. Students do need to meet a baseline proficiency level to access the content, but Christian Virtual School teachers are responsible for helping students develop their English literacy skills no matter the course they are enrolled in.

Upon enrollment, students are asked if they would like to provide information about their English language background, and this information is used by our teachers to help them adjust their instruction and suggest accommodations within the courses. English language learners are encouraged to reach out to their teacher or the Christian Virtual School administration to talk about the accommodation options in their courses so that the appropriate opportunities are given to everyone.

Environmental Education

Christian Virtual School operates with 5 cores values: responsibility, perseverance, integrity, compassion, and community. These core values determine our business operations, as well as exemplify what we, as educations, want to instill in our students. Environmental education, among other causes, are important to us as a school and we strive to promote learning about these issues and solutions within our courses. We work to educate students on the environment, its threats, and the importance of sustainability. We also work to inspire students to make an impact within their community and identify an alignment between their passions and the local, or global, needs.

Environmental education is woven throughout our course content, across all disciplines. Depending on the course and subject matter, this education can be subtle or explicit, but the goal is to ensure that students have the opportunity to acquire the knowledge, skills, perspective and practices needed to become an environmentally literate citizen.

Equity and Inclusive Education

Christian Virtual School stands on the belief that every person is unique and, regardless of ancestry, culture, ethnicity, sex, physical or intellectual ability, race, religion, sexual orientation, socio-economic status, or other similar factor, they are to be welcomed, included, accepted, treated fairly, and respected. As a school, we teach students about multiple worldviews, how to identify and acknowledge similarities and differences, and how to communicate with others in an inclusive, kind, loving, and compassionate way.

Diversity is valued at Christian Virtual School, and it is our goal to ensure all members of the community feel safe, comfortable, and accepted. Our courses are written to draw attention to the contributions of men and woman alike, the different perspectives of various cultural, religious, and racial communities, and the beliefs and practices of First Nations, Métis, and Inuit peoples, to showcase a wide range of backgrounds and allow all of our students to see themselves reflected in the curriculum.

As a school, we see and recognize the diversity of families, children, and people in the world in need of Christ's love. We work every day to spread the love and acceptance of Christ.

Financial Literacy Education

Whenever possible, Christian Virtual School emphasizes the importance of financial literacy. Making financial decisions has become an increasingly complex task, and students need to have knowledge in

many areas and a wide range of skills in order to make informed decisions about financial matters. In addition to the concrete skills of numeracy and finances from a mathematical point of view, students need to develop an understanding of the economic forces and ways in which they can respond to those influences.

Lessons that promote skill building in problem solving, inquiry, research, decision making, reflection, and critical thinking are present throughout Christian Virtual School courses. The goal is to help students acquire the knowledge and skills required to understand their own finances, as well as to develop an understanding of local and global effects of world economic forces and the social, environmental, and ethical implications of their own choices.

The Role of Information and Communication Technology

Technology is rapidly changing, and the requirements for literacy in technology is growing just as quickly. Students entering the workforce are expected to have a firm grasp of information and communication technologies and be skilled their use.

Due to the nature of Christian Virtual School courses, students are exposed to a wide range of technologies to both facilitate and communicate their learning. As a result, students will develop transferable skills through their experience with word processing, information processing, internet research, presentation software, communication tools, and more.

Career Education

Opportunities are present throughout Christian Virtual School courses to explore careers related to the different disciplines and subject areas. Students are exposed to a wide variety of modern careers, fields of study, and employment opportunities.

In addition, teachers are available to help the student prepare for employment in a number of diverse areas. With the help of teachers, students will learn to set and achieve goals and gain experience in making meaningful decisions concerning career choices. The skills, knowledge, and creativity that students acquire through our course are essential for a wide range of careers.

Health and Safety

In order to provide a suitable learning environment for the Christian Virtual School staff and students, it is critical that the courses and the learning environment complies with relevant federal, provincial, and municipal health and safety legislation and by-laws, including, but not limited to, the Workplace Safety and Insurance Act, the Workplace Hazardous Materials Information System (WHMIS), the Food and Drug Act, the Health Protection and Promotion Act, the Ontario Building Code, and the Occupational Health and Safety Act (OHSA).

Consideration of students' health and safety is taken when planning activities, investigations, and experiments for our courses to ensure that proper safety precautions are communicated to and attainable for students.