

Advanced Functions, Grade 12, University

Course Title: Advanced Functions

Course Code: MHF4U

Grade: 12

Course Type: University Preparation

Credit Value: 1.0

Prerequisites: MCR3U, Functions, or MCT4C, Mathematics for College Technology

Curriculum Document: [The Ontario Curriculum, Grade 11 and 12, 2007, Revised](#)

Developed By: Sarah McKercher

Department: Mathematics

Development Date: November 2023

Most Recent Revision Date: November 2023

Teacher(s):

Paul Leistra B.Sc., B.Ed., M.A., M.M.T., [OCT](#)

Course Description:

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

Unit Title and Description	Time Allocated
<p>Basic Skills Review</p> <p>This unit reviews the foundational concepts that have been covered in prerequisite math courses. Students revisit the definition of a function, function notation, and the key properties of functions. Students also review transformations of functions and inverse functions. The unit assessment evaluates students' ability to carry out proper communication, formatting, and technical skills in their work, all of which will be important aspects of their assignments in the remainder of the course.</p>	6 Hours
<p>Polynomial Functions</p> <p>In this unit students learn to identify and describe some key features of polynomial functions and to make connections between the numeric, graphical, and algebraic representations of polynomial functions. These concepts allow students to manipulate functions in a number of ways and apply their skills to solve real-world problems. Strategies will be employed to aid in the connection to an understanding of rates of change.</p>	20 Hours

<p>Rational Functions and Inequalities</p> <p>Students begin this unit by identifying and describing some of the key features of rational functions. Students then learn to represent and manipulate these functions to solve real-life problems, graphically and algebraically. This unit also introduces the idea of inequalities and how they produce different solutions than equations.</p>	15 Hours
<p>Exponential and Logarithmic Functions</p> <p>This unit begins with a review of exponential functions, their properties, and applications. This leads into discussions about a related function, the logarithmic function. From here students learn about logarithmic properties and then apply their knowledge of exponential and logarithmic functions to solve real-world problems.</p>	18 Hours
<p>Trigonometry</p> <p>This unit examines the meaning and application of radian measure. This allows students to solve more complex situations in exact values. Students will make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and use these connections to solve problems involving trigonometric equations and to prove trigonometric identities.</p>	18 Hours
<p>Trigonometry Functions and Graphs</p> <p>This unit develops students understanding of trigonometry by expanding on the functions behind the trigonometric ratios. Students look at trigonometric functions and their reciprocals, examine their key properties and behaviours, and learn how they can be transformed to model a wide range of data.</p>	18 Hours
<p>Operations and Functions</p> <p>Having studied various types of functions and transformations of functions, and understood the significance of differential rates of change in functions, this final unit focuses on the theory and practice of performing arithmetic operations on entire functions, including but not limited to the algebraic, graphical and practical implications of performing those operations.</p>	12 Hours
<p>Exam</p> <p>This is a proctored exam worth 30% of your final grade.</p>	3 Hours

Overall Curriculum Expectations

Exponential and Logarithmic Functions

1. demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify numeric expressions;

2. identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;
3. solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

Trigonometric Functions

1. demonstrate an understanding of the meaning and application of radian measure;
2. make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;
3. solve problems involving trigonometric equations and prove trigonometric identities.

Polynomial and Rational Functions

1. identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
2. identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;
3. solve problems involving polynomial and simple rational equations graphically and algebraically;
4. demonstrate an understanding of solving polynomial and simple rational inequalities.

Characteristics of Functions

1. demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
2. determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;
3. compare the characteristics of functions, and solve problems by modelling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

Resources Required:

This course is entirely online and does not require nor rely on any textbook. The materials required for the course are:

- A scanner, smart phone camera, or similar device to digitize handwritten or hand-drawn work
- A non-programmable, non-graphing, scientific calculator.
- Spreadsheet software (e.g. Microsoft Excel®, Apple Numbers®, Google Sheets®, or equivalent)
- Word processing software (e.g. Microsoft Word®, Apple Pages®, Google Docs®, or equivalent)
- Presentation software (e.g. Microsoft PowerPoint®, Apple Keynote®, Google Slides®, or equivalent)

- Graphing software (e.g. Desmos® graphing calculator) Other links may be provided in the course

Teaching and Learning Strategies:

The goal for this course is to help students use the language of mathematics skillfully, confidently and flexibly. To accomplish this, a wide variety of instructional strategies are used to provide learning opportunities to accommodate a variety of learning styles, interests, and ability levels. There are seven mathematical processes outlined in the Ontario curriculum that support effective learning in mathematics: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. These processes are used throughout the course as strategies for teaching and learning. The following list outlines their application further.

- *Problem solving:* The course guides students toward recognizing opportunities to apply knowledge they have gained in previous courses or lessons to solve problems. The course encourages students to persevere in difficult situations, look for patterns, build concrete skills in problem solving, and use logical reasoning to solve new problems.
- *Reasoning and proving:* This course has an emphasis on investigation and critical thinking as students explore new topics. This gives students the chance to make predictions, provide evidence, and explore relationships as they are taught the different mathematical concepts and relationships.
- *Reflecting:* At the end of each unit is a chance for students to reflect on their own learning, determine where their strengths are and where they should review before continuing. This self-reflection is an important skill in mathematics, as it enhances students' problem solving skills. Students are encouraged to reflect on the reasonableness of their answers, the effectiveness of a chosen strategy, and their conclusions.
- *Selecting tools and computational strategies:* Throughout the course students are exposed to and encouraged to utilize different tools, manipulatives, and strategies that best suit their learning needs.
- *Connecting:* This course connects the concepts taught to real-world applications through the use of word problems, career applications, and investigations.
- *Representing:* Through the use of examples, practice problems, and solution videos, the course models various ways to demonstrate understanding, poses questions that require students to use different representations as they are working at each level of conceptual development – concrete, visual or symbolic, and allows individual students the time they need to solidify their understanding at each conceptual stage.
- *Communicating:* Proper use of symbols, vocabulary, and notations is modeled throughout the course, and students are taught to use the same precision in their communications with their teacher. In addition, through the use of discussions, this course offers students

the opportunity to share their understanding both in oral as well as written form with their peers.

Assessment and Evaluation Strategies

Every student attending Christian Virtual School is unique. We believe each student must have the opportunities to achieve success according to their own interests, abilities, and goals. Like the Ministry of Education, we have defined high expectations and standards for graduation, while introducing a range of options that allow students to learn in ways that suit them best and enable them to earn their diplomas. Christian Virtual School's Assessment, Evaluation, and Reporting Policy is based on seven fundamental principles, as outlined in the [Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools](#) document.

When these seven principles are fully understood and observed by all teachers, they guide the collection of meaningful information that helps inform instructional decisions, promote student engagement, and improve student learning. At Christian Virtual School, teachers use practices and procedures that:

- are fair, transparent, and equitable for all students;
- support all students, including those with special education needs, those who are learning English, and those who are First Nation, Métis, or Inuit;
- are carefully planned to relate to the curriculum expectations and learning goals and, as much as possible, to the interests, learning styles and preferences, needs, and experiences of all students;
- are communicated clearly to students and parents or guardians at the beginning of the school year or course and at other appropriate points throughout the school year or course;
- are ongoing, varied in nature, and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning;
- provide ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement; and
- develop students' self-assessment skills to enable them to access their own learning, set specific goals, and plan next steps for their learning.

For more information on Christian Virtual School's assessment and evaluation strategies, you can refer to our [Assessment, Evaluation, and Reporting Policy](#).

Program Planning Considerations

Each of our courses have been designed by a team of educators to create an environment infused with creativity, flexibility, choice, and variety, with the goal to help every student succeed. We also take into consideration several topics that span disciplines and ensure we incorporate these into each of our courses.

Program Planning Considerations

Students with Special Needs

Christian Virtual School is committed to ensuring that all students are provided with the learning opportunities and supports they require to succeed. Our courses are made to offer flexible, personalized learning experiences. By maintaining an asynchronous model, students can move through their courses at their own pace, ensuring they are able to take the time they need to understand concepts or work with their teacher if they hit roadblocks. Christian Virtual School courses also incorporate choice, allowing students to submit work in a variety of mediums or formats to communicate their ideas.

In addition to the flexibility built into the courses, Christian Virtual School will implement the accommodations that are listed in a student's Individual Education Plan (IEP) that are applicable to the online learning environment. In these cases, the learning expectations will be the same as or similar to the expectations outlined in the curriculum document but supports will be provided to help students achieve those expectations. Common accommodations in the environment are reducing the workload, simplifying tasks and materials, providing extra time for tests and exams, allowing scribing or the use of specialized equipment, and not deducting marks for spelling.

English Language Learners

Although all our courses are only offered in English at this time, Christian Virtual School welcomes students learning the English language. Students do need to meet a baseline proficiency level to access the content, but Christian Virtual School teachers are responsible for helping students develop their English literacy skills no matter the course they are enrolled in.

Upon enrollment, students are asked if they would like to provide information about their English language background, and this information is used by our teachers to help them adjust their instruction and suggest accommodations within the courses. English language learners are encouraged to reach out to their teacher or the Christian Virtual School administration to talk about the accommodation options in their courses so that the appropriate opportunities are given to everyone.

Environmental Education

Christian Virtual School operates with 5 core values: responsibility, perseverance, integrity, compassion, and community. These core values determine our business operations, as well as exemplify what we, as educators, want to instill in our students. Environmental education, among other causes, are important to us as a school and we strive to promote learning about these issues and solutions within our courses. We work to educate students on the environment, its threats, and the importance of sustainability. We also work to inspire students to make an impact within their community and identify an alignment between their passions and the local, or global, needs.

Environmental education is woven throughout our course content, across all disciplines. Depending on the course and subject matter, this education can be subtle or explicit, but the goal is to ensure that students have the opportunity to acquire the knowledge, skills, perspective and practices needed to become an environmentally literate citizen.

Equity and Inclusive Education

Christian Virtual School stands on the belief that every person is unique and, regardless of ancestry, culture, ethnicity, sex, physical or intellectual ability, race, religion, sexual orientation, socio-economic status, or other similar factor, they are to be welcomed, included, accepted, treated fairly, and respected. As a school, we teach students about multiple worldviews, how to identify and acknowledge similarities and differences, and how to communicate with others in an inclusive, kind, loving, and compassionate way.

Diversity is valued at Christian Virtual School, and it is our goal to ensure all members of the community feel safe, comfortable, and accepted. Our courses are written to draw attention to the contributions of men and woman alike, the different perspectives of various cultural, religious, and racial communities, and the beliefs and practices of First Nations, Métis, and Inuit peoples, to showcase a wide range of backgrounds and allow all of our students to see themselves reflected in the curriculum.

As a school, we see and recognize the diversity of families, children, and people in the world in need of Christ's love. We work every day to spread the love and acceptance of Christ.

Financial Literacy Education

Whenever possible, Christian Virtual School emphasizes the importance of financial literacy. Making financial decisions has become an increasingly complex task, and students need to have knowledge in many areas and a wide range of skills in order to make informed decisions about financial matters. In addition to the concrete skills of numeracy and finances from a mathematical point of view, students need to develop an understanding of the economic forces and ways in which they can respond to those influences.

Lessons that promote skill building in problem solving, inquiry, research, decision making, reflection, and critical thinking are present throughout Christian Virtual School courses. The goal is to help students acquire the knowledge and skills required to understand their own finances, as well as to develop an understanding of local and global effects of world economic forces and the social, environmental, and ethical implications of their own choices.

The Role of Information and Communication Technology

Technology is rapidly changing, and the requirements for literacy in technology is growing just as quickly. Students entering the workforce are expected to have a firm grasp of information and communication technologies and be skilled their use.

Due to the nature of Christian Virtual School courses, students are exposed to a wide range of technologies to both facilitate and communicate their learning. As a result, students will develop transferable skills through their experience with word processing, information processing, internet research, presentation software, communication tools, and more.

Career Education

Opportunities are present throughout Christian Virtual School courses to explore careers related to the different disciplines and subject areas. Students are exposed to a wide variety of modern careers, fields of study, and employment opportunities.

In addition, teachers are available to help the student prepare for employment in a number of diverse areas. With the help of teachers, students will learn to set and achieve goals and gain experience in making meaningful decisions concerning career choices. The skills, knowledge, and creativity that students acquire through our course are essential for a wide range of careers.

Health and Safety

In order to provide a suitable learning environment for the Christian Virtual School staff and students, it is critical that the courses and the learning environment complies with relevant federal, provincial, and municipal health and safety legislation and by-laws, including, but not limited to, the Workplace Safety

and Insurance Act, the Workplace Hazardous Materials Information System (WHMIS), the Food and Drug Act, the Health Protection and Promotion Act, the Ontario Building Code, and the Occupational Health and Safety Act (OHSA).

Consideration of students' health and safety is taken when planning activities, investigations, and experiments for our courses to ensure that proper safety precautions are communicated to and attainable for students.